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University of Bahrain

College of Information Technology
Department of Computer Science

ITCS253 Discrete Mathematics

First Semester 2011/2012
Exam #1 — One Hour

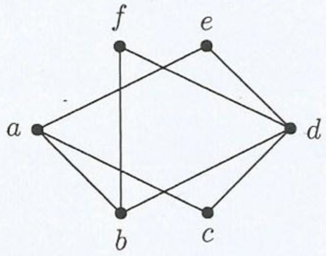
STUDENT NAME	DRAGON
STUDENT#	
SECTION	

Answer All Questions.
Make sure you have 4 pages.

QUESTION#	MARKS		COMMENTS
1	12	12	
2	12	12	
3	4	4	
4	6	6	
5	6	6	
TOTAL	40	40	

Instructors: Dr. Ali Alsaffar.
Dr. Ali Khan (Coordinator).

Q1. Consider the following graph.



(a) **[2 marks]** Is it a simple graph? Why?

2 yes, No loops and no multiple edges.

(b) **[2 marks]** Find the total degree of the graph. Can it be odd? Give reasons.

Total degree $= 2|E| = 2 \times 8 = 16$

2 It can't be odd because: $2 \times \text{even} = \text{even}$

$2 \times \text{odd} = \text{even}$

(c) **[2 marks]** Find a simple path from node c to b of length 4.

c d e a b

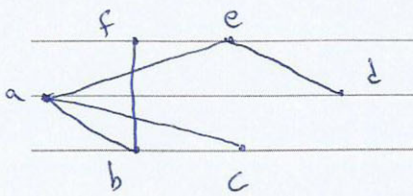
2

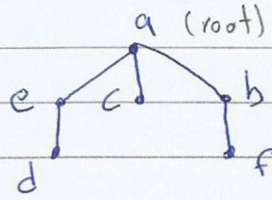
(d) **[2 marks]** Does the graph have Euler cycle? Give reasons.

No, because not all vertices have even degree

2 (deg(a) = 3)

(e) **[2 marks]** Find a spanning tree of the graph. Draw the tree rooted at vertex a.





2

spanning tree

spanning tree rooted at vertex a.

(f) **[2 marks]** Find adjacency matrix of the graph.

$$
\begin{array}{c|cccccc}
 & a & b & c & d & e & f \\
\hline
a & 0 & 1 & 1 & 0 & 1 & 0 \\
b & 1 & 0 & 0 & 1 & 0 & 1 \\
c & 1 & 0 & 0 & 1 & 0 & 0 \\
d & 0 & 1 & 1 & 0 & 1 & 1 \\
e & 1 & 0 & 0 & 1 & 0 & 0 \\
f & 0 & 1 & 0 & 1 & 0 & 0
\end{array}
$$

2

Q2. **[6 × 2 marks]** True or False. Justify your answer.

(1) ☑ True ☐ False A complete graph with more than two vertices is not bipartite.

because in a complete graph, there is an edge between any two nodes → there is a triangle between any three nodes.

2

(2) [☑ True ☐ False] It is not possible for a graph to have degree sequence 4, 4, 3, 3, 2, 2, 2, 1.

because total degree $= 4+4+3+3+2+2+2+1$

$= 21$ (odd)

it must be even.

2

(3) [☑ True ☐ False] The graph $K_{5,7}$ has 12 vertices and 35 edges.

$K_{5,7} \Rightarrow$ ~~V1~~ V1 contains 5 nodes and V2 contains 7 nodes

$\therefore n = 5+7 = 12$

no. of edges $= mn = 5\times 7 = 35$.

2

(4) [☐ True ☑ False] $K_{2,3}$ is isomorphic to K_5.

$K_{2,3} \Rightarrow$ no. of edges $= 2\times 3 = 6$

2 $K_5 \Rightarrow$ no. of edges $= \frac{n(n-1)}{2} = \frac{5(4)}{2} = 10$

$\therefore |E1| \neq |E2| \Rightarrow$ not isomorphic.

(5) [☐ True ☑ False] There exists a tree with 10 vertices such that the total degree is 24.

no. of edges in a tree $= n-1$, for $n=10$, $|E| = 9$

2

total degree $= 2|E| = 2\times 9 = 18 \neq 24$

(6) [☐ True ☑ False] A complete and full 3-*ary* tree with height 4 has 212 vertices.

$n = \frac{m^{h+1}-1}{m-1} = \frac{3^5-1}{3-1} = \frac{242}{2} = 121 \neq 212$

2

Q3. [**4 marks**] For a full *binary* tree with n vertices prove that it has $(n-1)/2$ internal vertices and $(n+1)/2$ leaves.

$\Rightarrow \frac{n+1}{2} + \frac{n-1}{2} = \frac{n+n+1-1}{2} = \frac{2n}{2} = n$

$n = mi+1$ ✓

$n = 2i+1$ ✓

$2i = n-1$ $\qquad n = i+L \Rightarrow L = n-i \Rightarrow i = n-L$ ✓

4 $\quad i = \frac{n-1}{2}$ ✓ $\Rightarrow n - L = \frac{n-1}{2}$

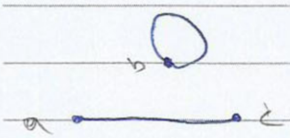
✓ $n - \frac{n-1}{2} = L \Rightarrow \frac{2n-n+1}{2} = L$

$= \frac{n+1}{2} = L$

Q4. [**6 marks**] Which of the following three matrices (if any) is the adjacency matrix of an undirected graph? In each case, either sketch the corresponding graph or explain why no such graph exists.

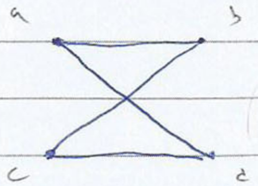
$$A_1 = \begin{array}{c} \\ a \\ b \\ c \end{array}\!\!\begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array} \qquad A_2 = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\!\!\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad A_3 = \begin{array}{c} \\ a \\ b \\ c \end{array}\!\!\begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$$

A1 is adjacency matrix for an undirected graph. (symmetric)



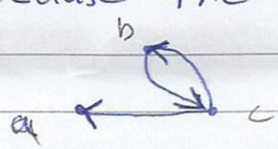
2

A2 is adjacency matrix for an undirected graph (symmetric)



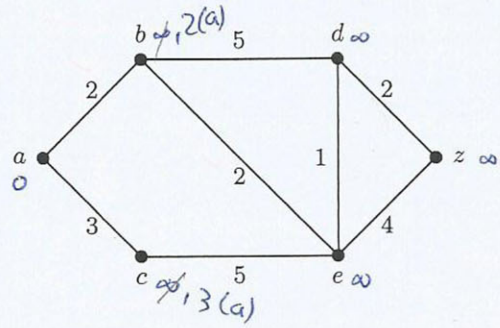
2

A3 is NOT adjacency matrix for an undirected graph because the matrix is not symmetrical



2

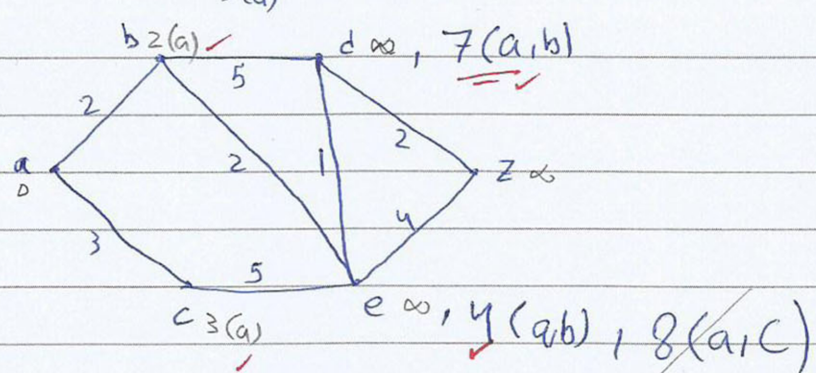
Q5. [6 marks] Find the shortest-path between a and z. Find the total weight of this path and list its edges from a to z. Show your work on the graph.



S= {a}

S= {a, b}

S= {a, b, C}



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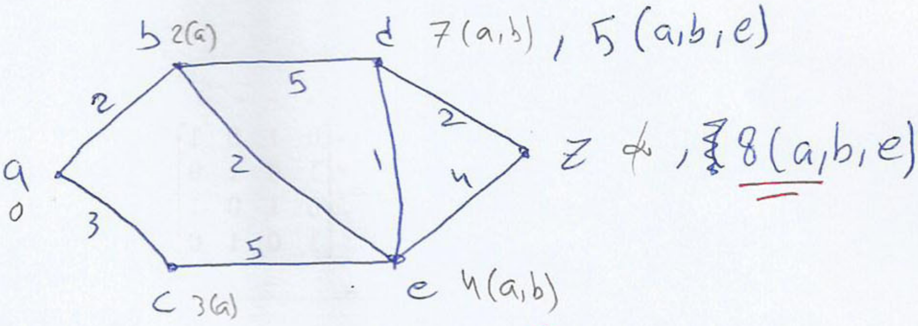
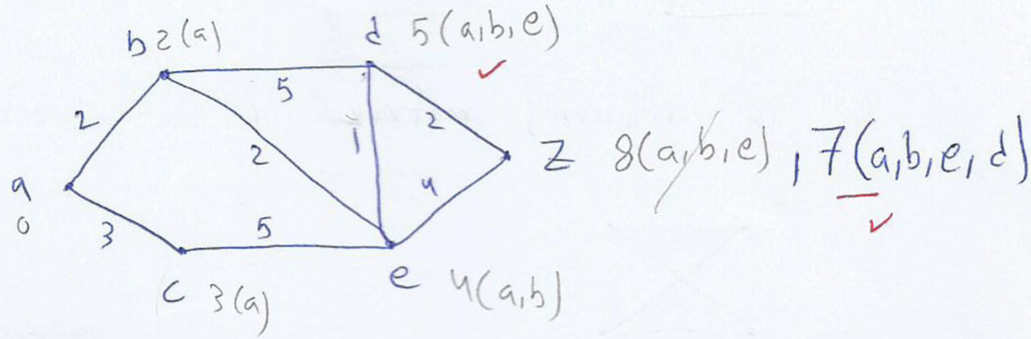


S = {a, b, c, e}



S = {a, b, c, e, d}

S = {a, b, c, e, d, Z}

∴ shortest path is : a b e d Z

of length = 7.